Calculating Soap Recipes

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I am new to soap making and wanted to understand how to build my own recipes for various soap molds. I did my web research and found many guidelines, thumb rules, and calculators that would help me do this, but still wanted a deeper understanding of what was behind the numbers and what it took to build a recipe from scratch.

In this paper, I offer the results of what I have learned. I go through a somewhat lengthy derivation of formulae for calculating with some precision the amount of each constituent in a soap recipe needed to fill a particular mold. At the end, I provide a sample calculation using the formulas to develop a recipe.

The Formulae for Filling a Mold

To start, the volume of each of the basic components of the soap recipe are added together to equal the volume of the mold:

$$V_{mold} = V_{oil} + V_{lyesoln} + V_{scent}$$

Where:

V_{mold} = Volume of the mold
V_{oil} = Total volume of oils
$V_{lyesoln}$ = Volume of the lye solution
V_{scent} = Volume of the scent oil

Then, each of these constituent volumes can then be separately derived.

Formula for V_{oil}

To start with, the volume of oil used is the sum of the individual volumes of each of the different oils used in the recipe and can be expressed as:

$$V_{oil} = V_{oil}^1 + V_{oil}^2 + \cdots + V_{oil}^n$$

Where:

V_{oil}^1 = Volume of the first oil of the recipe
V_{oil}^2 = Volume of the second oil of the recipe
... and so on until
V_{oil}^n = Volume of the last oil of the recipe

This can be written as:

$$V_{oil} = \sum_n V_{oil}^n$$

Where the symbol Σ is a shorthand for showing each of the individual oil volumes being added together.

The volume of each of the individual oils can also be expressed as the mass of the oil used divided by the density of the oil used.

$$V_{oil}^{n} = \frac{m_{oil}^{n}}{\rho_{oil}^{n}}$$

Where:

m_{oil}^{n} = Mass of the individual oil
ρ_{oil}^{n} = Density of the individual oil

The mass of the individual oils used can also be expressed as a fraction of the total mass of oil used or:

$$m_{oil}^{n} = f_{oil}^{n} m_{oil}$$

Where:

f_{oil}^{n} = Fraction of the individual oil (i.e., if the recipe calls for 25% of a particular oil, its fraction would be 0.25 or if it were 55% its fraction would be 0.55)
m_{oil} = Total mass of all oils used

With this, the volume of each individual oils can be expressed as:

$$V_{oil}^{n} = \frac{f_{oil}^{n} m_{oil}}{\rho_{oil}^{n}}$$

Then the total volume of all oil can be expressed as:

$$V_{oil} = \sum_{n} \frac{f_{oil}^{n} m_{oil}}{\rho_{oil}^{n}}$$

or, since m_{oil} , the total mass of all oils, is the same for each oil, this can be rewritten as:

$$V_{oil} = m_{oil} \sum_{n} \frac{f_{oil}^{n}}{\rho_{oil}^{n}}$$

This representation for the volume of oil will be used in a later calculation.

Formula for $\boldsymbol{V_{lyesoln}}$

Now moving on to the volume of the lye solution. I have seen some references state that the volume of the lye solution is simply the volume of the water used and lye used to create the solution. In fact, this is not the case. Depending on the concentration of the lye solution, this approach yields a volume for the lye solution that is as much as 15% different than the actual volume of the resultant solution. The formula developed here gives a more accurate volume. The volume of the lye solution can be represented as the mass (or weight) of lye solution divided by the density of the lye solution:

$$V_{lyesoln} = \frac{m_{lyesoln}}{\rho_{lyesoln}}$$

Where:

$m_{lyesoln}$ = Total mass of lye and water in the solution
$\rho_{lyesoln}$ = Density of the lye solution

The mass of lye solution is related to the mass of lye used by the fraction of the lye in the solution. This relationship can be represented as:

$$f_{lye} = \frac{m_{lye}}{m_{lyesoln}}$$

Where:

m_{lye} = Mass of lye
f_{lye} = Fraction of lye in solution (i.e., the fraction for a 30% lye solution would be 0.30)

or rewritten as

$$m_{lyesoln} = \frac{m_{lye}}{f_{lye}}$$

The mass of lye can be calculated based on the mass of each oil used and that oil's saponification value and the superfat percentage that is desired in the soap. This is as follows:

$$m_{lye} = \frac{1}{(1 + f_{sfat})}(S_{oil}^1\, m_{oil}^1 + S_{oil}^2\, m_{oil}^2 + \cdots + S_{oil}^n\, m_{oil}^n)$$

Where:

S_{oil}^1 = Saponification value of the first oil of the recipe
S_{oil}^2 = Saponification value of the second oil of the recipe
... and so on until
S_{oil}^n = Saponification value of the last oil of the recipe
m_{oil}^1 = Mass of the first oil of the recipe
m_{oil}^2 = Mass of the second oil of the recipe
... and so on until
m_{oil}^n = Mass of the last oil of the recipe
f_{sfat} = Superfat fraction (i.e., the fraction for a 5% superfat would be 0.05)

Again, in short hand, this can be written as:

$$m_{lye} = \frac{1}{(1 + f_{sfat})}\left(\sum_{n} S_{oil}^{n}\, m_{oil}^{n}\right)$$

Since we know from the oil volume formula that the mass of an individual oil can be represented by its fraction of the total oil, this becomes:

$$m_{lye} = \frac{1}{(1 + f_{sfat})}\left(\sum_{n} S_{oil}^{n}\, f_{oil}^{n} m_{oil}\right)$$

or, since m_{oil} is the same for each oil, this can be rewritten as:

$$m_{lye} = \frac{m_{oil}}{(1 + f_{sfat})}\left(\sum_{n} S_{oil}^{n}\, f_{oil}^{n}\right)$$

With this, $m_{lyesoln}$ becomes:

$$m_{lyesoln} = \frac{\frac{m_{oil}}{(1 + f_{sfat})}\left(\sum_{n} S_{oil}^{n}\, f_{oil}^{n}\right)}{f_{lye}}$$

or:

$$m_{lyesoln} = \frac{1}{f_{lye}}\frac{m_{oil}}{(1 + f_{sfat})}\sum_{n} S_{oil}^{n}\, f_{oil}^{n}$$

With this, $V_{lyesoln}$ becomes:

$$V_{lyesoln} = \frac{\frac{1}{f_{lye}}\frac{m_{oil}}{(1 + f_{sfat})}\sum_{n} S_{oil}^{n}\, f_{oil}^{n}}{\rho_{lyesoln}}$$

This representation for the volume of lye solution will be used in a later calculation.

Formula for V_{scent}

The last volume to calculate is for the volume of the scent. This is simply the mass or weight of the scent divided by the density of the scent or:

$$V_{scent} = \frac{m_{scent}}{\rho_{scent}}$$

Where:

m_{scent} = Mass of scent used
ρ_{scent} = Density of scent used

In general, the amount of scent to be used is based on the amount of oil used so this can be shown as:

$$m_{scent} = k_{scent} m_{oil}$$

Where:

k_{scent} = Selected ratio of scent to be used (i.e., ounces per pound of oils. Make sure to convert to consistent units for calculation)

The volume of scent oil used then becomes:

$$V_{scent} = \frac{k_{scent}}{\rho_{scent}} m_{oil}$$

This is the final volume value for the overall calculation of mold volume.

Putting it all together:

Recalling that:

$$V_{mold} = V_{oil} + V_{lyesoln} + V_{scent}$$

The individual volume formula for each of the constituent volumes can be substituted into this equation to yield a more complicated overall formula for what is needed to fill a particular mold volume:

$$V_{mold} = m_{oil} \sum_n \frac{f_{oil}^n}{\rho_{oil}^n} + \frac{\frac{1}{f_{lye}} \frac{m_{oil}}{(1 + f_{sfat})} (\sum_n S_{oil}^n f_{oil}^n)}{\rho_{lyesoln}} + \frac{k_{scent}}{\rho_{scent}} m_{oil}$$

or extracting out the common factor of m_{oil}:

$$V_{mold} = m_{oil} \left(\sum_n \frac{f_{oil}^n}{\rho_{oil}^n} + \frac{\frac{1}{f_{lye}} \frac{1}{(1 + f_{sfat})} (\sum_n S_{oil}^n f_{oil}^n)}{\rho_{lyesoln}} + \frac{k_{scent}}{\rho_{scent}} \right)$$

This equation can then algebraically be rewritten as an equation for m_{oil} to be:

$$m_{oil} = \frac{V_{mold}}{\sum_n \frac{f_{oil}^n}{\rho_{oil}^n} + \frac{\frac{1}{f_{lye}} \frac{1}{(1 + f_{sfat})} \sum_n S_{oil}^n f_{oil}^n}{\rho_{lyesoln}} + \frac{k_{scent}}{\rho_{scent}}}$$

This formula is the key for calculating all of the elements of a soap recipe. All of the elements on the right-hand side of the formula are either picked or looked up in tables:

Each f_{oil}^{n} is picked to total 100% when the recipe is crafted.

f_{lye} and f_{sfat} are picked when the recipe is crafted.

k_{scent} is picked when the recipe is crafted.

ρ_{oil}^{n}, $\rho_{lyesoln}$, and ρ_{scent} can be found in tables

S_{oil}^{n} can either be found in tables or data that is provided with an oil

Once m_{oil} is calculated using the formula, it can be used to calculate all of the recipe constituants

The mass of each individual oil can be calculated as:

$$m_{oil}^{n} = f_{oil}^{n} m_{oil}$$

The mass of lye can be calculated as:

$$m_{lye} = \frac{m_{oil}}{(1 + f_{sfat})} \sum_{n} S_{oil}^{n} f_{oil}^{n}$$

The mass of water can be calculated as:

$$m_{water} = \frac{(1 - f_{lye})}{f_{lye}} m_{lye}$$

And the mass of scent can be calculated as:

$$m_{scent} = k_{scent} m_{oil}$$

Example Recipe Calculation

In this example, I want to use the following as the basis for the recipe:

Mold size is 10” x 2.5” x 3.5”

The oil mix is to be:

65% Olive Oil
25% Coconut Oil
10% Almond Oil

Use a 3% superfat

Use a 30% lye solution

Use 0.65oz of Tuberose Scent per pound of oil (units of weight)

Calculating mold volume:

10” x 2.5” x 3.5” = 87.5 cubic inches

or

87.5 cubic inches x 16.3871 cc/cubic inch = 1433.9cc

Calculating the mass of oil is best done in steps. Remembering:

$$m_{oil} = \frac{V_{mold}}{\sum_n \frac{f_{oil}^n}{\rho_{oil}^n} + \frac{\frac{1}{f_{lye}} \frac{1}{(1+f_{sfat})} \sum_n S_{oil}^n f_{oil}^n}{\rho_{lyesoln}} + \frac{k_{scent}}{\rho_{scent}}}$$

it is helpful to make a table of the oil properties and tabulate the results of the calculations as follows:

Oil	Percent	f_{oil}^n	ρ_{oil}^n*	S_{oil}^n*	$\frac{f_{oil}^n}{\rho_{oil}^n}$	$S_{oil}^n f_{oil}^n$
Olive Oil	65%	0.65	0.9145	0.13515	0.71077	0.08785
Coconut Oil	25%	0.25	0.9171	0.18700	0.27260	0.04675
Almond Oli	10%	0.10	0.9140	0.13785	0.10941	0.01379
*From Tables				Sum	1.09278	0.14838

Also from tables:

$\rho_{lyesoln}$ for a 30% lye solution is 1.3277g/cc

ρ_{scent} for tuberose is 0.950g scent/cc

Converting k_{scent} to consistent units:

$$\frac{0.65oz\ scent}{pound\ oil} \times \frac{28.3495\text{g scent}}{oz\ scent} \times \frac{pound\ oil}{453.592\text{g oil}} = \frac{0.040625\text{g scent}}{\text{g oil}}$$

Substituting all of the values in the formula gives:

$$m_{oil} = \frac{1433.9\text{cc}}{1.09278\text{cc/g} + \frac{\frac{1}{0.30}\frac{1}{(1+0.03)}0.14838}{1.3277\text{g/cc}} + \frac{0.040625\text{g scent/g oil}}{0.950\text{g scent/cc}}}$$

Calculating yields:

$$m_{oil} = 957.6881\text{g}$$

The individual oils then by percentage are:

Olive Oil => $f_{oil}^{oliveoil} m_{oil}$ = 0.65 × 957.6881g = 622.4973g
Coconut Oil => $f_{oil}^{coconutoil} m_{oil}$ = 0.25 × 957.6881g = 239.4220g
Almond Oil => $f_{oil}^{almondoil} m_{oil}$ = 0.10 × 957.6881g = 95.7688g

The lye solution is:

Lye => $\frac{m_{oil}}{(1+f_{sfat})} \sum_n S_{oil}^n f_{oil}^n = \frac{957.6881\text{g}}{(1+0.03)}$ × 0.14838 = 137.965g

Water => $\frac{(1-f_{lye})}{f_{lye}} m_{lye} = \frac{(1-0.30)}{0.30}$ × 137.965g = 321.919g

The Scent is:

Scent => $k_{scent} m_{oil} = \frac{0.040625\text{g scent}}{\text{g oil}}$ × 957.6881g = 38.906g

All that remains is to weigh and mix.

Hopefully this was somewhat informative and of interest. I welcome your feedback.